6 SEM TDC DSE MTH (CBCS) 3 (H)

2023

(May/June)

MATHEMATICS

(Discipline Specific Elective)

(For Honours)

Paper : DSE–3

(Discrete Mathematics)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

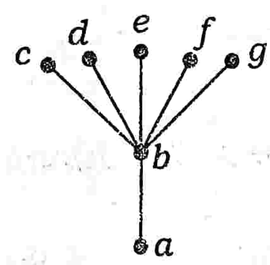
1. (a) Define an order-isomorphism. Is an order-isomorphism a bijective function? 1+1=2

   (b) Draw the Hasse diagram of the ordered set $(P(X), \subseteq)$ where $X = \{a, b, c\}$ and $P(X)$ is the power set of X. 3

(c) Let P and Q be two finite order sets. Then show that P and Q are order-isomorphic if and only if they can be drawn with identical Hasse diagram. 5

Or

Define the dual of an ordered set and state the duality principle. The ordered set P is given by the following Hasse diagram :



Find the dual of P. 2+1+2=5

2. (a) Define a lattice as an ordered set. Give an example. 1+1=2

(b) Let $(L, \leq)$ be a lattice and $a, b, c \in L$. Show that—

(i) $a \wedge (a \vee b) = a$;

(ii) $a \leq b$ and $a \leq c \Rightarrow a \leq b \wedge c$.

3+2=5

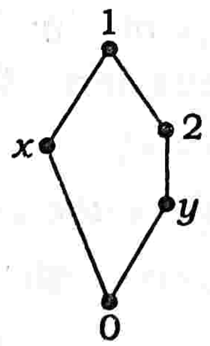
(c) Find three subsets of the lattice $(P(X), \subseteq)$ where $X = \{1, 2, 3\}$ and $P(X)$ is the power set of X, which are not sub-lattices of $(P(X), \subseteq)$. 3

(d) Define a complete lattice with an example. Let $(L, \vee, \wedge)$ be a complete lattice and 0 and 1 be the least and greatest elements of L respectively. Then show that $a \vee 0 = a$ and $a \vee 1 = 1$. 1+1+3=5

Or

Let $(L, \leq)$ be a lattice. Prove that L is a chain if and only if every non-empty subset of L is a sublattice. 5

3. (a) Show that the lattice pentagon depicted in the following Hasse diagram



is not modular. 2

(b) Let $(L, \vee, \wedge)$ be a distributive lattice and $a, b, c \in L$. If $a \wedge b = a \wedge c$ and $a \vee b = a \vee c$, then show that $b = c$. Hence show that complement of an element of L, if it exists, is unique. 3+1=4

(c) Let B be a Boolean algebra and $a, b \in B$. Then show that—

(i) $(a+b)' = a' \cdot b'$;

(ii) $(a')' = a$. 3+2=5

(d) Let $(A, \vee, \wedge)$ and $(B, +, \cdot)$ be two Boolean algebra and $f : A \to B$ be a function such that $f(a \vee b) = f(a) + f(b)$ and $f(a') = \overline{f(a)}$. Then show that f is a Boolean algebra homomorphism. 4

(e) Answer any *two* of the following : 5×2=10

(i) Obtain the sum-of-products canonical form of the Boolean expression

$$[\bar{x}_2 + \{\bar{x}_2 + x_1 + (\overline{x_2 x_3})\}]\,(x_2 + \bar{x}_1 x_2)$$

in the variables x_1, x_2 and x_3.

(ii) Obtain a minimal sum-of-products representation for the Boolean expression

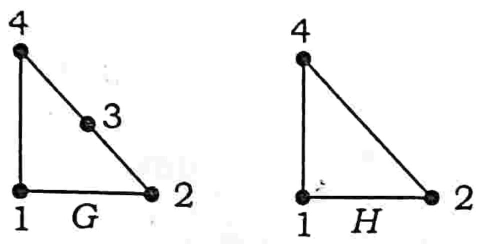
$$ab\bar{c} + abc + \bar{a}\bar{b}c + \bar{a}b\bar{c} + \bar{a}bc$$

using Karnaugh map.

(iii) Design a twin-switch that is used to switch on the light from the first step and the topmost step of a staircase, so that the light can be switched on/off using any of these switches. Give the logic network for this twin switches.

4. (a) Define a complete graph. Find the number of edges of the complete graph with 5 vertices. 1+1=2

(b) Graphs G and H are depicted in the following figure :



Is H a subgraph of G? 1

(c) Answer any *three* of the following : 3×3=9

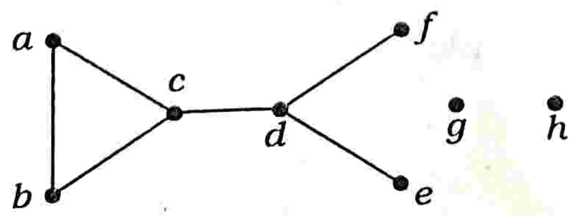
(i) Show that a bipartite graph cannot contain an odd cycle.

(ii) Draw the graph of K_7, $K_{3,4}$ and $K_{2,6}$.

(iii) Show that if a graph has exactly two odd vertices, then there exists a path between the two odd vertices.

(iv) Represent the Königsberg bridge problem by a graph. Does the Königsberg bridge problem have solution? Justify.

(d) Find the adjacency matrix of the graph. 2



 (Turn Over)

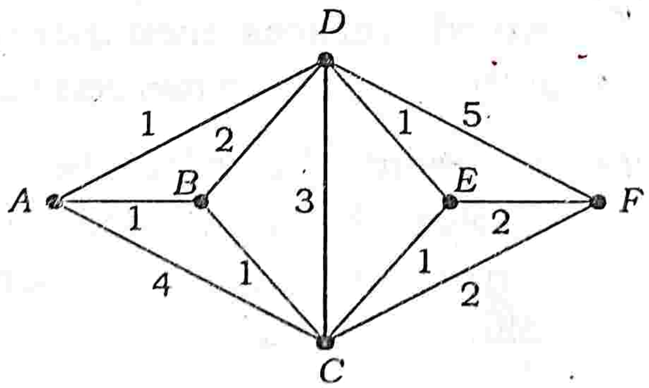
(e) A connected graph has 14 vertices and 88 edges. Show that G is Hamiltonian but not Eulerian. 2

5. Answer any *two* of the following : 7×2=14

(a) If a graph G has $n \geq 3$ vertices and every vertex has degree at least $\frac{n}{2}$, then show that G is Hamiltonian. 7

(b) Define Eulerian graph and Hamiltonian graph. Give an example of an Eulerian graph with 6 vertices which is not Hamiltonian. Show that if a graph G is Eulerian, then every vertex of G is of even degree. 1+1+1+4=7

(c) Find the distance between every pair of vertices of the following weighted graph using Floyd-Warshall algorithm : 7



(d) Discuss Dijkstra's algorithm with an example. 7

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