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6 SEM TDC DSE MTH (CBCS) 3 (H)

2022

(June/July)

MATHEMATICS

(Discipline Specific Elective)

(For Honours)

Paper : DSE–3

(Discrete Mathematics)

Full Marks : 80

Pass Marks : 32

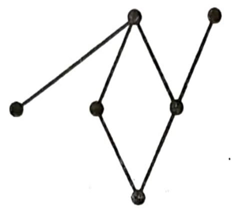
Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* Define an ordered set with an example. 1+1=2

(b) Show that the sets $\mathbb{N} \cup \{0\}$ and $\mathbb{N}$, where $\mathbb{N}$ is the set of natural numbers, are order-isomorphic. 3

(c) Let the ordered set P be given by the following Hasse diagram :



Find the dual of P. 2

(d) What is an order-preserving map? Give an example of a map which is order-preserving but not order-embedding. 1+2=3

2. (a) Answer any *one* of the following : 3

(i) Let P be a lattice and a, b, $c \in P$. Then show that $a \leq b \Rightarrow a \vee c \leq b \vee c$ and $a \wedge c \leq b \wedge c$.

(ii) Let L be a lattice and a, $b \in L$. Then show that $a \vee (a \wedge b) = a$.

(b) Find two non-trivial sublattices of the lattice $(P(X), \subseteq)$, where $X = \{1, 2, 3\}$ and $P(X)$ denotes the power set of X. 2

(c) Define a lattice isomorphism. 1

(d) Let L and K be two lattices and let $f: L \to K$ be a map. Show that the map f is a lattice isomorphism if and only if it is an order-isomorphism. 4

Or

Define a complete lattice. Show that every finite lattice is complete. 1+3=4

(e) Answer any *one* of the following : 5

(i) Let X be a set and Ω be a family of subsets of X, ordered by inclusion such that for every non-empty family $\{A_i\}_{i \in I} \subseteq \Omega$, the intersection $\bigcap_{i \in I} A_i \in \Omega$ and $X \in \Omega$. Then show that $(\Omega, \subseteq)$ is a complete lattice.

(ii) Let L be a lattice. Then prove that L is a chain if and only if every non-empty subset of L is a sublattice.

3. *(a)* Give an example of—

(i) a modular lattice which is not distributive;

(ii) a lattice which is neither modular nor distributive. 1+1=2

(b) Show that in a distributive lattice complement of an element, if exists, is unique. 3

(c) Let L be a Boolean lattice. Show that

$$(a \vee b)' = a' \wedge b'$$
$$(a \wedge b)' = a' \vee b' \ \forall \ a, b \in L$$ 4

(d) Let B and C be two Boolean algebras and $f : B \to C$ be a lattice homomorphism. Then show that the following are equivalent : 4

(i) $f(0) = 0$ and $f(1) = 1$

(ii) $f(a') = (f(a))' \ \forall \ a \in B$

(e) Obtain the sum-of-product canonical form of the Boolean polynomial $(x_1 + x_2)x_3$ in the variables x_1, x_2 and x_3. 2

(f) Answer any *two* of the following : 5×2=10

(i) Show that a lattice L is non-modular if and only if L has a sublattice isomorphic to the pentagon lattice N_5.

(ii) Obtain a minimal expression for the Boolean polynomial

$$ab'c' + abc' + a'b'c + a'bc'$$

using Karnaugh map.

(iii) Obtain a minimal expression for the Boolean polynomial

$$abc + abc' + a'bc' + a'b'c$$

using Quine-McCluskey algorithm.

4. (a) Define a connected graph. 1

(b) Find the number of edges of the graph $K_{8,10}$. 2

(c) Answer any *three* of the following : 3×3=9

(i) Show that in a graph the number of odd vertices is even.

(ii) Show that the maximum number of edges in a connected simple graph with n vertices is $\frac{n(n-1)}{2}$.

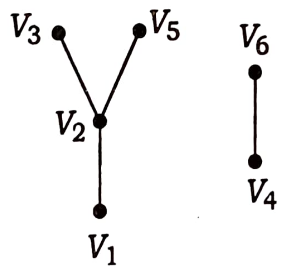
(iii) State three necessary conditions for isomorphism of two graphs.

(iv) Show that a bipartite graph cannot contain an odd cycle.

(d) Prove or disprove : 2

If a graph G is Eulerian, then G is also Hamiltonian.

(e) Find the adjacency matrix of the graph shown below : 2



5. Answer any *two* of the following : 7×2=14

(a) Show that a connected graph G is Eulerian if and only if every vertex of G is even.