# Total No. of Printed Pages—20 5 SEM TDC DSE MTH (CBCS) 1.1/1.2/1.3 (H)

### 2022

### (Nov/Dec)

### MATHEMATICS

(Discipline Specific Elective)

(For Honours)

Paper : DSE-1

Full Marks : 80 Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

Paper : DSE-1.1

## (Analytical Geometry)

1. Answer the following questions :

(a) Write the vertex of the conic

$$(x-1)^2 = 2(y+2)$$

Find the equation of the ellipse whose ends of major axis  $(0, \pm 6)$ , and passes through the point (-3, 2).

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## (2)

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Write the processes to sketch the ellipse.

(d) Identify and sketch the curve

$$y^2 - 8x - 6y - 23 = 0$$

and also label the focus, vertex and directrix.

Or

Describe the graph of the hyperbola

$$16x^2 - u^2 - 32x - 6y - 57 = 0$$

and sketch its graph.

2. Answer the following questions :

(a) Write the condition of tangency of the line y = mx + c to the parabola  $y^2 = 4ax$ .

(b) Write the reflection property of ellipse. 1

(c) Write the equation of the asymptotes of the hyperbola  $\frac{x^2}{4} - \frac{y^2}{9} = 1.$ 

(d) Derive the equation of tangent to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  at the point  $(x_1, y_1)$ .

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# (3)

(e) Find the equation of the hyperbola whose length of transverse axis 7 units and foci (±5, 0) and also sketch it.

#### Or

Find and sketch the curve of the ellipse whose foci (1, 2) and (-1, -2) and the sum of the distances from each point P(x, y)on the ellipse is 6 units.

3. Answer the following questions :
(a) Write the condition that the equation ax<sup>2</sup> + 2hxy + by<sup>2</sup> + 2gx + 2fy + c = 0 represent a pair of straight lines.

(b) Write the condition that the quadratic equation

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

represents an ellipse.

(c) Determine a rotation angle  $\theta$  that will eliminate the xy-term of the conic

$$x^2 - 4xy + 4y^2 - 5 = 0 2$$

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(d) Show that the graph of the given equation

$$x^2 - 10\sqrt{3}xy + 11y^2 + 64 = 0$$

is a hyperbola. Find its foci, vertices and asymptotes.

(e)

Let an x'y'-coordinate system be obtained by rotating an xy-coordinate system through an angle  $\theta = 60^{\circ}$ .

(i) Find the x'y'-coordinate of the point whose *xy*-coordinate is (–2, 6).

(ii) Find an equation of the curve  $\sqrt{3}xy + y^2 = 6$  in x'y'-coordinate.

Or Identify and sketch the curve  $9x^2 - 24xy + 16y^2 - 80x - 60y + 100 = 0$ 

- 4. Answer the following questions :
  - Write the equation of a sphere whose .(a) centre is at the origin and radius is r.
  - (b) Write True or False : Curve of intersection of two spheres is a

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Write the standard equation of hyperbola of one sheet.



Write the equation of the tangent plane to the sphere

$$x^{2} + y^{2} + z^{2} + 2ux + 2vy + 2wz + d = 0$$

at  $P(x_1, y_1, z_1)$ .



() Find the equation of the sphere passes through the points (0, 0, 0), (0, 1, -1), (-1, 2, 0), (1, 2, 3).

A sphere of constant radius k passes (f)through the origin and meets axes in A, B and C. Prove that the centroid of the triangle ABC lies on the sphere

$$9(x^2 + y^2 + z^2) = 4k^2$$
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#### Or

Find the equation of the sphere whose centre at (1, 2, 3) and touching a plane at (2, 1, 3).

5. Answer the following questions :

(a) Find the radius and centre of the circle  $x^{2} + y^{2} + z^{2} - 8x + 4y + 8z - 45 = 0, x - 2y + 2z = 3$ 

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(b) Find the equation of the sphere whose great circle is  $x^{2}+y^{2}+z^{2}+10y-4z-8=0, x+y+z=3$ Or

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Prove that the two spheres  $x^{2} + y^{2} + z^{2} - 2x + 4y - 4z = 0$ and  $x^{2} + y^{2} + z^{2} + 10x + 2z + 10 = 0$ touch each other.

- 6. Answer the following questions :
  - (a) Find the equation of the two tangent planes to the sphere

 $x^{2} + y^{2} + z^{2} - 2y - 6z + 6 = 0$ 

which are parallel to the plane

$$2x + 2y - z = 0$$

(b) Classify and sketch the quadric surface (any one) : (i)  $36x^2 + 9y^2 + 16z^2 = 144$ (ii)  $4x^2 - 3y^2 + 12z^2 + 12 = 0$ 

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